Assignment 7: Sums, Limits, and Taylor Polynomials
Week 11

[Note: We had a late Spring Break, and an unseasonably warm week. I needed to get the remaining assignments of the semester on the students' radar. Hence, I provided this overview of coming attractions.]

Mid-Semester Reflections: DUE 10:00 pm Sunday, [date]

You have received mid-semester feedback from me – both your midterm grade and a summary of the grades I have recorded for you in this course. We have one month (15 class sessions) remaining this semester. This is a good time to review the course objectives, and to be sure that you will be able to meet these objectives by the end of the semester. Please review the course objectives, which were listed on the Syllabus, and think about your own reasons for taking this course. Please send me your responses to the following questions by 10 pm on Sunday [date]. I will summarize your reflections and respond to your feedback in class on Monday, [Day 31]. It would be possible to make some changes in how we use class time for the remaining weeks of the semester based on your feedback.

Considering the course objectives and the expectations you had when you signed up for this course:

1. What is your sense of the progress you are making in meeting the course objectives?
2. What aspects of the course do you feel are working well for you? What do you feel is working poorly?
3. Which (one or two) of the assignments or class activities has been the most helpful for you in terms of understanding and meeting the course objectives? In what way has this assignment or project been helpful?
4. Identify two or three things that you can do which would help you to meet the course objectives.
5. In what way(s) could Sister Barbara help you to do the things you identify in item 4?
6. Do you have any suggestions for changes that we could try – in the way we use class time, in the class activities and homework assignments, in any other aspect of the course – which might help to make this an even better course for you?

Please type up your responses in Word, and upload this document to your folder for this course in Educator. Send me your responses to these questions by 10 pm on Sunday April 11. Please consider this an individual assignment, not a group assignment. I am interested in hearing what each of you has to say about how this course is going – and how we can make it even better.

<u>An overview of the rest of the semester</u>:

Chapter 10: Polynomial and Series Representations of Functions

This chapter will bring together two topics that we have been using informally throughout the entire course (both last semester in Calculus I and this semester in Calculus II): *sums* and *limits*. We have been using ideas about limits when we calculate derivatives, and let the h or the Δx in the denominator "go to zero." We have used sums as we've worked with polynomial functions, and in setting up a sum of areas of rectangles (or trapezoids) to approximate the area bounded by a curve and the x-axis. We have used ideas about limits when we talk about setting up a Riemann sum and letting Δt go to 0 while n goes to ∞.

A *polynomial* is a sum. A *series* is like a polynomial except that it has an infinite number of terms. The problem is to determine whether a series adds up to a finite number (that is, whether it converges), or adds up to ∞ or to $-\infty$ (diverges). Series computations are important in physics and economics, as well as in writing programs for computers and calculators to represent trigonometric functions. To perform series computations, we will learn to do calculations involving infinite limits.

One more Benchmark: Offered in class on Monday, [Day 40]

The window-of-opportunity for passing this benchmark will be May 3 – 10. An explicit study guide for this Benchmark will be available by Monday, April 26. Topics to be covered on this Benchmark will be:

- Calculating derivatives, antiderivatives, and definite integrals, which we have been doing all semester (The Study Guide for Test 3 includes lists of functions for which you should be able to calculate derivatives and antiderivatives.)
- Evaluating limits, which is covered in Chapter 10

One more project:

Project 4 will be available on [Day 36] and due on [Day 41]. This project will make use of ideas that we are studying in chapter 10.

Cumulative Final Exam: Scheduled for Tuesday, [insert date and time of Final Exam]

An overview of the next three classes:

In class on Wednesday, [Week 11, Day 29]:

- Lecture over important ideas of Section 10.1: Sums and Limits
- Group work on Activities and Checkpoints of Section 10.1
- Homework: Study Section 10.1, and complete any the Activities and Checkpoints if you don't finish them in class.

In class on Friday, [Day 30]:

- Lecture over important ideas of Section 10.2: Approximation of Functions: Taylor Polynomials
- Group work on Activities and Checkpoints of Section 10.2
- Homework: Study Section 10.2, and complete any the Activities and Checkpoints if you don't finish them in class.

In class on Monday, [Day 31]:

- More examples and discussion of selected exercises from Sections 10.1 and 10.2.

Class notes for Assignment 7 [Days 29 – 31]: Sums, Limits, and Taylor Polynomials

Mid-Semester Reflections: DUE 10:00 pm [evening before Day 29]

- I will be passing back tests and projects at the end of class.
- Your reflections and constructive feedback could change the way that we use class time.

<u>An overview of the rest of the semester</u>:

Chapter 10: Polynomial and Series Representations of Functions

This chapter brings together two topics that we have been using informally throughout the entire course: *sums* and *limits*. Reminders about:

- Derivatives are defined as the limit of a difference quotient
- Polynomials are sums
- Definite integrals are "areas," and we've calculated approximate definite integrals by setting up Riemann sums – then thinking about what happens as $\Delta t \to 0$ and $n \to \infty$
- A *series* is like a polynomial except that it has an infinite number of terms. The problem is to determine whether a series adds up to a finite number (that is, whether it converges), or adds up to ∞ or to $-\infty$ (diverges).

One more Benchmark: Offered in class on [Day 40]

The window-of-opportunity for passing this benchmark will be [Days 40 – 43]. An explicit study guide for this Benchmark will be available by [Day 37].

One more project:

Project 4 will be available on [Day 36] and due on [Day 41]. This project will make use of ideas that we are studying in chapter 10.

Cumulative Final Exam: Scheduled for [Day, date, and time for Final Exam]

<u>An overview of the next three classes</u>:

In class on [Day 29]:

- Lecture over important ideas of Section 10.1: Sums and Limits
- Section 10.1.1: Geometric Sums
  - Outcomes of coin-tossing experiments – making a table, calculating probabilities
  - Walk through Activity 1, concept of geometric sums, and Example 1
  - Notation for limits
  - Calculations / formula for Newton's Method for finding roots
  - Finding limits (Checkpoint 2) – use any ideas you have: calculate some values, think about the graphs, guess-and-explain …

- Group work on Activities and Checkpoints of Section 10.1.1
- Section 10.1.2: More Notation for Limits
  - Walk through Example 2 – making connections to the formula developed in 10.1.1
  - Walk through Example 3 – making connections to differentiation, and developing a new limit formula

- Group work on Activities and Checkpoints of Section 10.1.2
- <u>Homework</u>: Study Section 10.1, and complete any of the Activities and Checkpoints you didn't finish in class.

In class on [Day 30]:

- Lecture over important ideas of Section 10.2: Approximation of Functions: Taylor Polynomials
  - Quick review of polynomials: A polynomial is a sum of power functions.

- Taking derivatives of polynomials

- Group work on Activity 1, Example 1, and Checkpoint 1 of Section 10.2
- Walk through Example 2, making connections to their observations from Example 1 and Checkpoint 1: We are constructing a polynomial to approximate the exponential function.
- Taylor polynomials (and Maclaurin polynomials)

- Group work on Activity 2 of Section 10.2.3: Constructing the Taylor polynomial for sin(x)
- Homework: Study Section 10.2, and complete any the Activities and Checkpoints if you don't finish them in class.

In class on [Day 31]:
Summary of strategies for calculating limits:

- If there is a "zero in the denominator" you can often fix the problem algebraically. If you can eliminate the zero in the denominator, you have an expression that is identical to the original expression – except at the problem point.
- A sketch graph of the function will often help us to "see" what is going on in the neighborhood of the problem points. You should be able to calculate limits by using a sketch graphs.
- In order to compute the limit of an expression as the variable goes to infinity, it often helps to think about what is happening as the variable gets larger (goes toward $+\infty$) or gets smaller (goes toward $-\infty$).
- You should be able to calculate limits for constant functions, linear functions, polynomial functions, absolute value functions, and rational functions.
- In Section 10.1.2, we saw ways of using what we know about calculating derivatives to develop additional rules for limits as we need them.

More examples and discussion of selected exercises from Sections 10.1 and 10.2